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JOURNAL BEARING VELOCITY PROFILES FOR SMALL ECCENTRICITY AND MODERATE MODIFIED REYNOLDS NUMBERS

by

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ABSTRACT

The velocity profiles for an infinite, cylindrical bearing are obtained by means of a small eccentricity perturbation calculation. The modified Reynolds number appears as a parameter, and velocity profiles are presented for modified Reynolds numbers of 10^{-3}, 10^{-2}, 10^{-1}, 1, 10 and 10^{2}. The most significant difference in the velocity profiles for the various Reynolds numbers is the appearance of components which are 90° out of phase with the film thickness at the larger values of the modified Reynolds number. The consequences of these components are discussed.

INTRODUCTION

The standard technique for treating hydrodynamic lubrication problems involves a truncation of the Navier-Stokes equations. The truncation is based on the ratio of bearing clearance to bearing radius and on the modified Reynolds number both being small. The smallness of clearance to radius ratio is used to justify the neglect of derivatives along the bearing surface compared to derivatives normal to the surface, and the smallness of the modified Reynolds number justifies the neglect

[redacted] ... situations have arisen in recent

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years, however, in which the modified Reynolds number is of unit order or larger. The standard approximations cannot be justified in these situations, and experience has shown that the performance predictions based on these approximations are inadequate[1,2].

The significant changes in journal bearing characteristics at increased modified Reynolds numbers may be attributed to three phenomenological changes[3]. First, the flow may remain laminar and inertial contributions merely become important. Secondly, the flow may remain laminar but be radically altered by the onset of a secondary flow, eg. Taylor vortices. And finally, the flow may become turbulent.

The possibility of the inertial effects becoming significant at moderate values of the modified Reynolds number has been analytically investigated by two methods[4]. One method amounts to an iteration technique. The velocity and pressure profiles are first determined via the standard procedure of neglecting the inertial terms, and then a correction is calculated by introducing these velocity profiles into the inertial terms of the more complete equations. That is, if $\bar{p}_o$, $\bar{v}_o$, and $\bar{u}_o$ are the terms obtained by neglecting inertia, and $\bar{p}_1$, $\bar{v}_1$ and $\bar{u}_1$ are the correction terms, then the corresponding momentum equations in cartesian coordinates are

$$0 = \frac{\partial \bar{p}_o}{\partial \bar{x}} + \bar{\mu}\frac{\partial^2 \bar{u}_o}{\partial \bar{y}^2} \qquad \text{(A)}$$

and

$$\rho\left(\bar{u}_o \frac{\partial \bar{u}_o}{\partial \bar{x}} + \bar{v}_o \frac{\partial \bar{u}_o}{\partial \bar{y}}\right) = -\frac{\partial \bar{p}_1}{\partial \bar{x}} + \bar{\mu}\frac{\partial^2 \bar{u}_1}{\partial \bar{y}^2} \qquad \text{(B)}$$

Clearly, $\bar{u}_1$, $\bar{v}_1$ and $\bar{p}_1$ are equivalent to first order terms of perturbations expansions in powers of the modified Reynolds number,

$$u = u_o + Re^* u_1 + Re^{*2} u_2 + Re^{*3} u_3 + \ldots\ldots$$

$$v = v_o + Re^* v_1 + Re^{*2} v_2 + \ldots\ldots$$

$$p = p_o + Re^* p_1 + Re^{*2} p_2 + \ldots\ldots$$

The disconcerting fact is that Equations (A) and (B) have been used to calculate corrections for values of Re^* up to five, and the expansion procedure is only useful for $Re^* < 1$. For $Re^* > 1$ the expansions may even diverge because $\{u_1, u_2, \ldots.\}$, $\{v_1, v_2, \ldots.\}$ and $\{p_1, p_2, \ldots.\}$ are not known a priori to be rapidly diminishing sequences of functions. The other method of correcting for inertial effects is by averaging the convective acceleration terms in the momentum equations before solving for the velocity and pressure profiles. In this scheme the x momentum equation appears as

$$\bar{\rho}\left\{ \frac{1}{\bar{h}} \int_0^{\bar{h}} \left(\bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} \right) d\bar{y} \right\} = - \frac{\partial \bar{p}}{\partial \bar{x}} + \bar{\mu} \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \qquad \text{(C)}$$

where the term in braces is to be estimated before solving for $\bar{p}$ and $\bar{u}$. Some initial assumptions must be made regarding the forms of $\bar{u}$ and $\bar{v}$ in order to implement this scheme, and the procedure is to assume that $\bar{u}$ and $\bar{v}$ are similar to the profiles obtained by the total neglect of inertia. Neither of the preceding methods will admit velocity profiles which differ noticably from zero inertia profiles, and the first method cannot be justified for $Re^* > 1$. However, their results rule out the possibility of significant inertial effects, and this conclusion appears to be widely, if uncritically, accepted.

The possibility of a secondary flow regime existing in the bearing clearance has recently been discussed[3], and experiments[5,6] are presently being carried out to study the matter further. However, no calculations have been performed on the basis of a vortex regime to date, and no apparent effort is being made to formulate lubrication problems in terms of a vortex flow.

The idea of a transition to turbulence at moderate values of the modified Reynolds number had a strong intuitive appeal, and numerous calculations have been carried out on this basis[7,8,9,10]. The earlier references[7,8] imply an abrupt transition similar to that in pipes, while the latter papers[9,10] explicitly picture the transition region as extending over a narrow but significant range of modified Reynolds numbers. However, all of the turbulent treatments to date assume a similarity with turbulent duct flows and assume that the inertial terms may be neglected. These two assumptions are extensively discussed in reference 3, and it is concluded therein that both are doubtful at best.

It would thus appear that the foundations of high speed lubrication theory are far from satisfactory. The inertial terms for a laminar flow regime have been shown to be negligible by a calculation which inherently assumes that they are negligible. The formulation of a theory based on a vortex regime has never been attempted, and the existing turbulent theories introduce questionable physical assumptions. Clearly, the nature of the flow must be known before any progress can be made. The crux of ascertaining the flow regime is gaining an adequate knowledge of changes in the velocity profiles as the modified Reynolds number approaches and, perhaps, exceeds unity. This is the main goal of the present work.

DESCRIPTION OF THE PROBLEM

The task of determining the behavior of the velocity profiles for varying Reynolds numbers is rather formidable, and some care must be taken in selecting the proper approach. For one thing, the analysis must not preclude the possibility of changes in the basic characteristics of the profiles as the Reynolds number is increased. Such changes would actually be of primary interest. Moreover, the method of exhibiting the profiles' dependence on the modified Reynolds number must lead to tractable calculations, and a complete solution of the Navier-Stokes equations is much too ambitious a project. A possible means of achieving tractability without restricting the modified Reynolds number is by introducing solely geometric restrictions on the class of bearings to be considered.

Incompressible flow in an infinite, concentric, cylindrical journal bearing has been extensively studied by fluid mechanists, and the knowledge of the flow regime is rather complete for low to moderate values of the modified Reynolds number. The classical rotating Couette flow velocity profiles persist until the onset of Taylor vortices at a predictable Reynolds number. If the eccentricity were non-zero, but very small, the flow should not deviate much from that of the concentric case. Assuming this, the pre-vortex profiles for small eccentricity can be investigated by means of a perturbation scheme in which the perturbation parameter is related to the eccentricity. The only restriction on the Reynolds number is that it be less than the critical value for Taylor vortices. The behavior of the perturbation quantities as the Reynolds number varies from very low values, for which standard lubrication theory suffices, to the Taylor boundary will provide insight into the changes that occur in general lubrication profiles as the level of inertia is increased.

The perturbation scheme used in the following analysis is well suited to both a case in which the shaft center is space fixed and a particular case of full frequency whirl. Since the accepted method of analyzing shaft dynamics tacitly assumes a quasi-steady tangential velocity profile in the lubricant film, the solutions for an orbiting shaft are of considerable interest. A comparison of the perturbation velocities for a static and a dynamic shaft will provide some insight into the validity of this quasi-steady assumption. If a comparison shows marked differences between the perturbation profiles, this would indicate a stronger coupling between the lubricant film and the motion of the shaft than is presently assumed. Therefore, both cases will be considered in the following analysis.

The present investigation differs from previous work in the field of high-speed lubrication both in terms of goals and method. Previous work has been directly concerned with the calculation of performance parameters for a variety of practical bearing configurations. Numerous assumptions regarding the lubricant flow were used in carrying out these calculations, and the resulting performance predictions are dependent upon the assumptions. The present analysis seeks only an understanding of the fluid mechanic behavior of lubricant films as the Reynolds number is increased. In a sense, it is concerned with the validity of the assumptions made by previous authors.

It has already been indicated that the following analysis will be restricted to a simpler bearing geometry and to small eccentricties. Due to the small eccentricity restriction, the results can only be used directly in predicting performance over a narrow operating range. However, this limited range is of practical importance. Moderate values of the modified Reynolds number are to be anticipated in space applications where liquid metals are used as lubricants. Since reduced gravity conditions are part of the space environment, small eccentricities are then to be expected. Therefore, the restrictions to small eccentricities may not be as serious a practical limitation as the restriction on bearing geometry.

ANALYSIS

The configuration of interest is represented in figure (1). It consists simply of two circles whose centers are not quite coincident and, with a change in viewpoint, represents both the case in which the inner circle (the shaft) rotates with a space-fixed center and the case in which the center of the inner circle (the shaft) orbits about the center of the outer circle. In the first case, a space-fixed observer sees the inner circle rotating at a constant angular velocity about its center which is also fixed in space while the outer circle remains stationary. In the second case, the observer sits on the center of the inner circle while it orbits the center of the outer circle. To the observer it appears that the inner circle is fixed and the outer circle is rotating in the opposite direction with constant angular velocity. The latter case may be interpreted as a particular example of full frequency whirl.

The momentum equations for the space fixed and body fixed observers differ by the addition of apparent force terms in the latter case. However, the governing equation for the stream function is the same in both cases,* and it will be most convenient to work directly in terms of the stream function. The governing equation for the two cases is

$$\frac{\partial(\bar{\nabla}^2\bar{\Psi})}{\partial\bar{t}} + \frac{1}{\bar{r}}\frac{\partial(\bar{\nabla}^2\bar{\Psi},\bar{\Psi})}{\partial(\bar{r},\ \bar{\theta})} = \frac{1}{\nu}\bar{\nabla}^2\bar{\nabla}^2\bar{\Psi} \tag{1}$$

where $\bar{\nabla}^2$ is the usual two-dimensional Laplacian operator in polar coordinates and $\bar{\Psi}$ is defined such that

$$\bar{V}_r = \frac{1}{\bar{r}}\frac{\partial\bar{\Psi}}{\partial\bar{\theta}}$$

*This is easily seen by taking the curl of the apparent forces.

and

$$\bar{V}_\theta = -\frac{\partial \bar{\Psi}}{\partial \bar{r}}$$

(Equation (1) and the definition of $\bar{\Psi}$ imply that the fluid is incompressible and has constant viscosity.) The boundary conditions in the two cases are:

Case I

$$\left.\begin{aligned} \bar{V}_r &= 0 \\ \bar{V}_\theta &= \omega r_1 \end{aligned}\right\} \quad \text{at the inner circle}$$

$$\left.\begin{aligned} \bar{V}_r &= 0 \\ \bar{V}_\theta &= 0 \end{aligned}\right\} \quad \text{at the outer circle}$$

Case II

$$\left.\begin{aligned} \bar{V}_r &= 0 \\ \bar{V}_\theta &= 0 \end{aligned}\right\} \quad \text{at the inner circle}$$

$$\left.\begin{aligned} \bar{V}_r &= 0 \\ \bar{V}_\theta &= \omega r_2 \end{aligned}\right\} \quad \text{at the outer circle}$$

The cylindrical system, although conventional, is not a convenient coordinate system in which to treat the problem because the boundary conditions must be enforced on curves which are not coordinate lines. A more convenient system, in which the two circles do appear as coordinate lines, is achieved by the conformal transformation

$$\rho e^{i\phi} = \frac{re^{i\theta} + r_1(\delta - \varepsilon)}{\delta re^{i\theta} + r_1(1 - \delta\varepsilon)} \tag{2}$$

where

$$\delta \equiv -2\varepsilon/\left\{\frac{r_2^2}{r_1^2} - 1 - \varepsilon^2 + \left(\frac{r_2^2}{r_1^2} - 1 - \varepsilon^2\right)^2 - 4\varepsilon^2\right\}^{1/2} \tag{3}$$

In the ρ,ϕ system the inner circle transforms into the coordinate curve $\rho = 1$ and the outer circle transforms into the curve $\rho = \beta$ where

$$\beta \equiv r_2/r_1(1 - \delta\varepsilon) \tag{4}$$

This type of transformation has previously been used in similar problems by Wannier[11], Wood[12] and Segel[13].

A stream function, ψ , can also be introduced in the ρ,ϕ system. It is related to Ψ by the equation

$$\overline{\nabla}^2\Psi = \overline{J}\nabla^2\overline{\psi} \tag{5}$$

where $\overline{\nabla}^2$ is the two-dimensional Laplacian operator in the r,θ system, ∇^2 is the two-dimensional Laplacian operator in the ρ,ϕ and $\overline{J}$ is the Jacobian of the coordinate transformation,

$$\bar{J} = r_1^2(1 - \delta^2)^2/(1 - 2\delta\rho\cos\phi + \rho^2\delta^2)^2 \qquad (6)$$

The velocity components in the ρ and ϕ directions respectively are

$$\bar{u} = \frac{r_1^2}{\rho\bar{J}^{1/2}}\left(\frac{\partial\bar{\psi}}{\partial\phi}\right)$$

and

$$\bar{v} = -\frac{r_1^2}{\bar{J}^{1/2}}\left(\frac{\partial\bar{\psi}}{\partial\rho}\right)$$

If we non-dimensionalize by letting*

$$t = \omega\bar{t}\ ; \qquad J = \bar{J}/r_1^2\ ; \qquad \psi = \bar{\psi}/\omega$$

Equation (1) can be rewritten as

$$\frac{\partial(\nabla^2\psi)}{\partial t} + \frac{1}{\rho}\frac{\partial(\nabla^2\psi/J,\psi)}{\partial(\rho,\phi)} = \frac{1}{Re}\nabla^2(\nabla^2\psi/J) \qquad (7)$$

where $Re \equiv r_1^2\omega/\nu$. The boundary conditions on ψ are:

Case I

$$\left.\frac{\partial\psi}{\partial\phi}\right|_{\rho=1} = 0\ ; \qquad \left.\frac{\partial\psi}{\partial\phi}\right|_{\rho=\beta} = 0$$

$$\left.\frac{\partial\psi}{\partial\rho}\right|_{\rho=1} = -J^{1/2}(1,\phi)\ ; \quad \left.\frac{\partial\psi}{\partial\rho}\right|_{\rho=\beta} = 0$$

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*While it is physically clear that $\bar{\psi}$ and $\bar{J}$ are correctly scaled by ω and r_1^2, a time scale connected with ν would probably be more appropriate for $\bar{t}$. However, it turns out that the scale for $\bar{t}$ is of no significance in the present analysis.

Case II

$$\left.\frac{\partial\psi}{\partial\phi}\right|_{\rho=1} = 0 \; ; \qquad \left.\frac{\partial\psi}{\partial\phi}\right|_{\rho=\beta} = 0$$

$$\left.\frac{\partial\psi}{\partial\rho}\right|_{\rho=1} = 0 \; ; \qquad \left.\frac{\partial\psi}{\partial\rho}\right|_{\rho=\beta} = \left(\frac{r_2}{r_1}\right) J^{1/2}(\beta,\phi)$$

Now that the boundary value problem has been formulated, the next step is to motivate the perturbation scheme. First, we note that δ, which is essentially the ratio of eccentricity to radial clearance, and r_2/r_1 are the only geometric parameters appearing in the problem (δ through the Jacobian). The only reason for the appearance of (r_2/r_1) is the desire for treating the two cases (fixed eccentricity and rotating eccentricity) simultaneously, and it can be eliminated by using r_2 rather than r_1 as the length scale for the case with rotating eccentricity. Therefore, the appearance of r_2/r_1 is somewhat artificial. However, δ is an important geometric parameter which characterizes the problem being considered. If the eccentricity goes to zero, $\delta = 0$, $J = 1$, and the problem reduces to that of rotating Couette flow. Moreover, if $\delta \neq 0$ but $\delta << 1$, Equation (7) and the boundary conditions are not quite satisfied by the Couette solutions because J and $J^{1/2}$ depart from unity by terms of order δ. Therefore, if the governing equation and boundary conditions for ψ are expanded and ordered in powers of δ, the solution for rotating Couette flow will clearly satisfy the lowest-order problem. This indicates the feasibility of attempting a solution of the form

$$\psi = [\psi_o(\rho) + \delta\psi_1(\rho,\phi,t) + \delta^2\psi_2(\rho,\phi,t) + \delta^3\psi_3(\rho,\phi,t) + \ldots] \tag{8}$$

for sufficiently small δ. The ψ_n are assumed to be of unit order, and ψ_o is the well-known stream function for rotating Couette flow,

$$\psi_o = A\rho^2 + B \ln \rho$$

where

<u>Case I</u> -- fixed eccentricity

$$A = 1/2(\beta^2 - 1) \; ; \qquad B = -\beta^2/(\beta^2 - 1)$$

<u>Case II</u> -- rotating eccentricity

$$A = (1/2)(r_2/r_1)\beta/(\beta^2 - 1) \; ; \qquad B = -(r_2/r_1)\beta/(\beta^2 - 1)$$

In the standard way then, Equation (8) is substituted into Equation (7), the resulting differential equation and boundary conditions are expanded in powers of δ, and the coefficients of $\delta, \delta^2, \ldots$ are individually set equal to zero. Consequently, the following sequence of linear boundary value problems is obtained.

$O(\delta)$

$$\frac{\partial\nabla^2\psi_1}{\partial t} - \frac{1}{\rho}\frac{\partial\psi_o}{\partial\rho}\frac{\partial}{\partial\phi}\left(\nabla^2\psi_1 - 16A\rho\cos\phi\right) = \frac{1}{Re}\nabla^2\nabla^2\psi_1 \tag{9a}$$

Case I

$$\left.\frac{\partial\psi_1}{\partial\phi}\right|_{\rho=1} = \left.\frac{\partial\psi_1}{\partial\phi}\right|_{\rho=\beta} = \left.\frac{\partial\psi_1}{\partial\rho}\right|_{\rho=\beta} = 0$$

$$\left.\frac{\partial\psi_1}{\partial\rho}\right|_{\rho=1} = -2\cos\phi$$

Case II

$$\left.\frac{\partial\psi_1}{\partial\phi}\right|_{\rho=1} = \left.\frac{\partial\psi_1}{\partial\phi}\right|_{\rho=\beta} = \left.\frac{\partial\psi_1}{\partial\rho}\right|_{\rho=1} = 0$$

$$\left.\frac{\partial\psi_1}{\partial\rho}\right|_{\rho=\beta} = 2\beta^2\cos\phi$$

$O(\delta^2)$

$$\frac{\partial\nabla^2\psi_2}{\partial t} - \frac{1}{\rho}\frac{\partial\psi_o}{\partial\phi}\frac{\partial}{\partial\rho}\left(\nabla^2\psi_2 - 4\rho\cos\phi\nabla^2\psi_1 + 8A\rho^2\cos 2\phi\right) =$$

$$\frac{1}{Re}\nabla^2\left(\nabla^2\psi_2 - 4\rho\cos\phi\,\nabla^2\psi_1 + 8A\rho^2\cos 2\phi\right) +$$

$$[-\frac{64A}{Re} + (\frac{1}{\rho}\frac{\partial\psi_1}{\partial\phi}\frac{\partial\nabla^2\psi_1}{\partial\rho} - \frac{\partial\psi_1}{\partial\rho}\frac{1}{\rho}\frac{\partial\nabla^2\psi_1}{\partial\phi})] \qquad (9b)$$

Case I

$$\left.\frac{\partial\psi_2}{\partial\phi}\right|_{\rho=1} = \left.\frac{\partial\psi_1}{\partial\phi}\right|_{\rho=\beta} \quad \left.\frac{\partial\psi_1}{\partial\rho}\right|_{\rho=\beta} = 0$$

$$\left.\frac{\partial\psi_2}{\partial\rho}\right|_{\rho=1} = -2\cos 2\phi$$

Case II

$$\left.\frac{\partial\psi_2}{\partial\phi}\right|_{\rho = 1} = \left.\frac{\partial\psi_2}{\partial\phi}\right|_{\rho = \beta} = \left.\frac{\partial\psi_2}{\partial\rho}\right|_{\rho = 1} = 0$$

$$\left.\frac{\partial\psi_2}{\partial\phi}\right|_{\rho = \beta} = 2\beta^3 \cos 2\phi$$

$O(\delta^3)$ ......

In the following we shall confine our attention to the first-order problem, i.e. the boundary value problem for ψ_1. The velocity profiles corresponding to ψ_1 provide the most significant deviation from ψ_o. Thus, the effect of the Reynolds number on ψ_1 will be the most important. The perturbation velocities corresponding to $\psi_2, \psi_3 \ldots$ are of higher order in δ and are primarily of interest in studying the perturbation scheme itself.

At this point the details of the analysis of the two cases differ sufficiently to make separate considerations advisable. They will be reunited, however, for discussion of the quasi-steady assumption.

Case I -- Fixed Eccentricity:

Since Equation (9a) is linear and the boundary conditions for ψ_1 are independent of time, ψ_1 can be represented as a sum of two terms,

$$\psi_1 = F(\rho,\phi) + G(\rho,\phi,t)$$

where F satisfies the equation

$$-\frac{1}{\rho}\frac{\partial\psi_o}{\partial\rho}\frac{\partial}{\partial\phi}\left(\nabla^2 F - 16A\rho\cos\phi\right) = \frac{1}{Re}\nabla^2\nabla^2 F \tag{10}$$

and the boundary conditions

$$\left.\frac{\partial F}{\partial\phi}\right|_{\rho=1} = \left.\frac{\partial F}{\partial\phi}\right|_{\rho=\beta} = \left.\frac{\partial F}{\partial\rho}\right|_{\rho=\beta} = 0$$

$$\left.\frac{\partial F}{\partial\rho}\right|_{\rho=1} = -2\cos\phi$$

and G satisfies the equation

$$\frac{\partial\nabla^2 G}{\partial t} - \frac{1}{\rho}\frac{\partial\psi_o}{\partial\rho}\frac{\partial}{\partial\phi}\nabla^2 G = \frac{1}{Re}\nabla^2\nabla^2 G$$

and a set of homogeneous boundary conditions.

The boundary value problem for G has a simple physical interpretation. The eigensolutions for G correspond to the possible infinitesimal circumferential waves or Tollmien-Schlicting disturbances in rotating Couette flow. The neutral boundary for circumferential waves was investigated by Tamaki and Harrison(14) and Harrison(15) before Taylor's(16) original work on the vortex regime was published, and more recent work on the circumferential waves has been done by DiPrima(17) and DiPrima and Stuart(18). The latter investigations permit the simultaneous existence of the vortex modes, for it is now well known that the critical Reynolds number for circumferential disturbances is greater than the critical Reynolds number for Taylor vortices. Since the present analysis is restricted to Reynolds numbers less than the critical value for Taylor vortices, the eigensolutions for G will all contain damping factors and have little influence on the present steady-flow problem. Therefore, G will be neglected altogether.

A most significant feature of lubrication problems has not yet been incorporated into the boundary value problem for F. That is, the thinness of the lubricant film has not been introduced yet. This can be done formally by introducing the new variables

$$\rho = (1 + \Delta y) \quad ; \quad \Delta = \beta - 1$$

$$\phi = x$$

$$F(\rho,\phi) = 2\Delta\eta(y,x)$$

and the modified Reynolds number

$$Re^* = \Delta^2 Re$$

The correct lubrication approximation to Equation (10) is easily deduced by substituting x,y and η into Equation (10) and then retaining only the lowest order terms as $\Delta \to 0$. This corresponds to the familiar small gap limit in rotating Couette flow stability analyses. The resulting equation and boundary conditions on η are

$$(1 - y) \frac{\partial}{\partial x} \left[\frac{\partial^2\eta}{\partial y^2} - 2 \cos x\right] = \frac{1}{Re^*} \frac{\partial^4\eta}{\partial y^4} \qquad (11)$$

$$\left.\frac{\partial\eta}{\partial x}\right|_{y=0} = \left.\frac{\partial\eta}{\partial x}\right|_{y=1} = \left.\frac{\partial\eta}{\partial y}\right|_{y=1} = 0$$

$$\left.\frac{\partial \eta}{\partial y}\right|_{y = 0} = - \cos x$$

Equation (11) and the boundary conditions on η at $y = 0,1$ constitute the boundary value problem we will solve. The simplest procedure is first to rewrite Equation (11) as a homogeneous equation in terms of the variable $\gamma = \eta - y^2 \cos x$

$$(1 - y) \frac{\partial}{\partial x} \left(\frac{\partial^2 \gamma}{\partial y^2}\right) = \frac{1}{Re^*} \frac{\partial^4 \gamma}{\partial y^4} \tag{12}$$

Equation (12) admits solutions of the form $\gamma = e^{ix} f(y)$. Consequently, we arrive at a fourth-order ordinary differential equation for f ,

$$f^{IV} - iRe^*(1 - y) f'' = 0 \tag{13}$$

where f is a complex function of y, $(f(y) = f_R(y) + i f_I(y))$. Equation (13) is then rewritten in terms of the variables $\xi = (1 - y) Re^{*1/3}$ and $g(\xi) = f(y)$

$$g^{IV} - i \xi g'' = 0 \tag{14}$$

to remove the parameter Re^* . Equation (14) is a Bessel's equation for g'' , and the general solution for g'' can be written down directly. The most convenient representation is

$$g''(\xi) = \xi^{1/2} \{C_1 J_{1/3}(2/3\, e^{3\pi i/4}\, \xi^{3/2}) +$$

$$C_2 J_{-1/3}(2/3\, e^{3\pi i/4}\, \xi^{3/2})\} \tag{15}$$

where

$$J_\nu(z) = \sum_{n=o}^{\infty} \frac{(-1)^n (z/2)^{\nu+2n}}{n!\Gamma(\nu + n + 1)}$$

is the standard Bessel function of the first kind. Equation (15) can be rewritten as

$$g'' = C_1 \sum_{n=o}^{\infty} A_n \xi^{3n+1} + C_2 \sum_{n=o}^{\infty} B_n \xi^{3n} \tag{16}$$

where

$$A_n \equiv \frac{(-1)^n \; e^{(6n+1)\pi i/4}}{3^{(6n+1)} n!\Gamma(n + 4/3)}$$

and

$$B_n \equiv \frac{(-1)^n \; e^{(6n-1)\pi i/4}}{3^{(6n-1)} n!\Gamma(n + 2/3)}$$

Integrating term by term we get

$$g' = C_1\beta_1(\xi) + C_2\beta_2(\xi) + C_3 \tag{17}$$

and

$$g = C_1\alpha_1(\xi) + C_2\alpha_2(\xi) + C_3\xi + C_4 \tag{18}$$

where

$$\beta_1(\xi) \equiv \sum_{n=o}^{\infty} A_n \xi^{(3n+2)}/(3n + 2) \quad ,$$

$$\beta_2(\xi) \equiv \sum_{n=o}^{\infty} B_n \xi^{3n+1}/(3n + 1) \quad ,$$

$$\alpha_1(\xi) \equiv \sum_{n=o}^{\infty} A_n \xi^{3n+3}/(3n + 2)(3n + 3) \quad ,$$

and

$$\alpha_2(\xi) \equiv \sum_{n=o}^{\infty} B_n \xi^{3n+2}/(3n + 1)(3n + 2) \quad .$$

The boundary conditions on $g(\xi)$, obtained from the conditions on η , are that

$$g(0) = 1 \quad , \quad g(Re^{*1/3}) = 0 \quad ,$$

$$g'(0) = -2Re^{*-1/3} \quad \text{and} \quad g'(Re^{*1/3}) = -Re^{*-1/3}$$

It should be borne in mind that g and g' are complex functions, and the boundary conditions on η are derived from physical restrictions on the velocities. Therefore, the boundary conditions on g and g' are such that only the real parts of η need satisfy the boundary conditions.

The four constants, C_1, C_2, C_3 and C_4 must now be determined by satisfying the boundary conditions on g . Since α_1, α_2, β_1 and β_2 are all zero at $\xi = 0$, C_3 and C_4 can be written down immediately,

$$C_3 = -\frac{2}{Re^{*1/3}} \quad ; \quad C_4 = 1$$

The remaining two constants are to be determined by solving the following pair of simultaneous equations

$$C_1\alpha_1(Re^{*1/3}) + C_2\alpha_2(Re^{*1/3}) = 1 \tag{19}$$

$$C_1\beta_1(Re^{*1/3}) + C_2\beta_2(Re^{*1/3}) = Re^{*-1/3} \tag{20}$$

C_1, C_2, $g(\xi)$ and $g'(\xi)$ have been numerically determined for $Re^* = 10^{-3}, 10^{-2}, 10^{-1}, 1, 10, 10^2$. Rather than merely listing $g(\xi)$ and $g'(\xi)$ though, it is more meaningful to rearrange the results into a form that can be more readily interpreted. For this purpose we return to the velocity components, $\bar{u}$ and $\bar{v}$. By definition

$$\bar{u} = \frac{\omega r_1}{\rho J^{1/2}} \frac{\partial \psi}{\partial \phi} = \frac{\omega r_1(1 - 2\delta\rho \cos\phi + \rho^2\delta^2)}{(1 - \delta^2)\rho}\left[\frac{\partial \psi_o}{\partial \phi} + \delta \frac{\partial \psi_1}{\partial \phi} + \delta^2 \frac{\partial \psi_2}{\partial \phi} + \ldots\right]$$

and

$$\bar{v} = -\frac{\omega r_1}{J^{1/2}} \frac{\partial \psi}{\partial \rho} = -\frac{\omega r_1(1 - 2\delta\rho \cos\phi + \rho^2\delta^2)}{(1 - \delta^2)}\left[\frac{\partial \psi_o}{\partial \rho} + \delta \frac{\partial \psi_1}{\partial \rho} + \ldots\right]$$

and to first order in δ

$$\bar{u} \approx \frac{\omega r_1 \delta}{\rho} \frac{\partial \psi_1}{\partial \phi}$$

and

$$\bar{v} \approx -\omega r_1\left[\frac{\partial \psi_o}{\partial \rho} + \delta\left(\frac{\partial \psi_1}{\partial \rho} - 2\rho \cos\phi \frac{\partial \psi_o}{\partial \rho}\right)\right]$$

By introducing the lubrication approximation next, which retains only the lowest order terms in Δ , this reduces to

$$\bar{u} \approx 2\omega r_1 \Delta\delta \frac{\partial\eta}{\partial x} = 2\omega r_1 \Delta\delta\left[\left(g_R(\xi) - (1 - Re^{*-1/3}\xi)^2\right)\sin x + g_I(\xi) \cos x\right] \tag{21}$$

and

$$\bar{v} \approx \omega r_1(1 - y) - 2\omega r_1 \delta\left[\frac{\partial\eta}{\partial y} + (1 - y) \cos x\right] = \omega r_1 Re^{*-1/3}\xi - 2\omega r_1 \delta\left[\left(Re^{*1/3} g_R'(\xi) + 2 - Re^{*-1/3}\xi\right)\cos x - Re^{*1/3} g_I'(\xi) \sin x\right] \tag{22}$$

Again, the only restriction on Re^* in Equations (21) and (22) is that it be less than the critical value for Taylor vortices. The bracketed terms in Equations (21) and (22) are the lowest order eccentricity perturbation velocities within the lubrication approximation, and the leading term in Equation (22) is simply the lubrication approximation to the rotating Couette flow profile. Only the bracketed terms are of interest to us. Figures 2 through 5 show curves of $\left(g_R(\xi) - (1 - Re^{*-1/3}\xi)^2\right)$, $g_I(\xi)$, $\left(Re^{*1/3} g_R'(\xi) + 2 - Re^{*-1/3}\xi\right)$ and $Re^{*1/3} g_I'(\xi)$ versus $\xi Re^{*-1/3}$ with the modified Reynolds number as a parameter.

<u>Case II -- Rotating Eccentricity</u>

The procedure for obtaining solutions for Case II is analogous to Case I, and we need only sketch out the steps. We first set

$$\psi_1 = H(\rho,\phi) + I(\rho,\phi,t)$$

where H satisfies the equation

$$-\frac{1}{\rho}\frac{\partial\psi_o}{\partial\rho}\frac{\partial}{\partial\phi}\left(\nabla^2 H - 16A\rho\cos\phi\right) = \frac{1}{Re}\nabla^2\nabla^2 H \qquad (23)$$

and the boundary conditions,

$$\left.\frac{\partial H}{\partial\rho}\right|_{\rho=1} = \left.\frac{\partial H}{\partial\phi}\right|_{\rho=\beta} = \left.\frac{\partial H}{\partial\rho}\right|_{\rho=1} = 0$$

$$\left.\frac{\partial H}{\partial\rho}\right|_{\rho=\beta} = 2\beta^2\cos\phi$$

I corresponds to G and need not be discussed. By introducing the lubrication scaling

$$\rho = (1+\Delta y) \quad ; \quad \Delta = \beta - 1$$
$$\phi = x$$

and an analogous set of substitutions for H,

$$H = 2\Delta\left(e^{-ix}h(y) + y^2\cos x\right)$$

$$\zeta = yRe^{*1/3}$$

and

$$k(\zeta) = -h(y)$$

we get an equation identical to (14)

$$k^{IV} - i\zeta k'' = 0 \qquad (24)$$

Clearly then

$$k'(\zeta) = D_1\beta_1(\zeta) + D_2\beta_2(\zeta) + D_3 \qquad (25)$$

and

$$k(\zeta) = D_1\alpha_1(\zeta) + D_2\alpha_2(\zeta) + D_3\zeta + D_4 \qquad (26)$$

The boundary conditions on $k(\zeta)$ are

$$k(0) = 0 \quad ; \quad k(Re^{*1/3}) = 1$$

$$k'(0) = 0 \quad ; \quad k'(Re^{*1/3}) = Re^{*-1/3}$$

Since $\alpha_1(0)$, $\alpha_2(0)$, $\beta_1(0)$ and $\beta_2(0)$ are all zero, it follows from the boundary conditions at $\zeta = 0$ that

$$D_3 = D_4 = 0$$

and from the remaining two boundary conditions we get the simultaneous equations

$$D_1\alpha_1(Re^{*1/3}) + D_2\alpha_2(Re^{*1/3}) = 1 \qquad (27)$$

$$D_1\beta_1(Re^{*1/3}) + D_2\beta_2(Re^{*1/3}) = Re^{*-1/3} \qquad (28)$$

By comparing Case I and Case II we see that

$$C_1 = D_1 \quad \text{and} \quad C_2 = D_2$$

and therefore,

$$g(\xi) = k(\xi) - 2Re^{*-1/3}\xi + 1 \qquad (29)$$

and

$$g'(\xi) = k'(\xi) - 2Re^{*-1/3} \tag{30}$$

If we now go through similar steps to obtain velocity components, we find that

$$\bar{u} \approx 2\omega r_1 \delta\Delta[\left(k_R(\zeta) - Re^{*-2/3}\zeta^2\right)\sin x + k_I(\zeta) \cos x] \tag{31}$$

and

$$\bar{v} \approx -\omega r_1 Re^{*-1/3}\zeta + 2\omega r_1 \delta[\left(Re^{*1/3}k_R'(\zeta) - Re^{*-1/3}\zeta\right)\cos x + Re^{*1/3}k_I'(\zeta) \sin x] \tag{32}$$

The terms of interest then are $\left(k_R(\zeta) - Re^{*-2/3}\zeta^2\right)$, $k_I(\zeta)$, $\left(Re^{*1/3}k_R'(\zeta) - Re^{*-1/3}\zeta\right)$ and $Re^{*1/3}k_I'(\zeta)$.

<u>Comparison of Cases I and II</u>

The main idea of this section is to compare the velocity components for Cases I and II, thereby checking the quasi-steady assumption to within the order of our approximation. Equations (29) and (30) tell us that the imaginary parts of g and k and the imaginary parts of g' and k' are identical. Therefore, the $\bar{u}$ term multiplied by $\cos x$ and $\bar{v}$ term multiplied by $\sin x$ are functionally the same for both fixed and rotating eccentricity. Furthermore, by use of Equation (29) we find that

$$g_R(\xi) - (1 - \xi Re^{*-1/3})^2 = k_R(\xi) - (\xi Re^{*-1/3})^2$$

and by use of Equation (30) we find that

$$Re^{*1/3} g_R'(\xi) + 2 - Re^{*-1/3}\xi = Re^{*1/3} k_R'(\xi) - Re^{*-1/3}\xi$$

Therefore, the $\bar{u}$ component multiplied by $\sin x$ and $\bar{v}$ component multiplied by $\cos x$ are also functionally the same in both cases. This means that the quasi-steady assumption is valid within the thin film and small eccentricity approximation. Due to the above results, all the information for Case II is already contained in figures 2 through 5.

There is one more significant point to be considered in comparing the two cases. It was not explicitly stated, but the arguments regarding the function G in the fixed eccentricity case can also be applied to the function I in the rotating eccentricity case. This means that the rotating observer in Case II is no more likely to see time dependent motions than the space fixed observer in Case I. The main influence of the oscillating boundary (Case II as seen by a space fixed observer) must be connected with the way in which disturbances propagate in a flow which is itself periodic.

DISCUSSION

In this section we shall interpret the results. To do this meaningfully it should always be borne in mind that the main objective is to determine the changes that take place in the velocity profiles as the modified Reynolds number or global level of inertia is increased.

To make the interpretation somewhat clearer we will begin by obtaining the small eccentricity perturbation profiles within the classical lubrication limit, i.e. $Re^* << 1$. Equation (11) for the limiting case $Re^* \to 0$ is

$$\frac{\partial^4 \tilde{\eta}}{\partial y^4} = 0$$

where $\tilde{\eta}$ denotes the limit of η as $Re^* \to 0$. The boundary conditions on $\tilde{\eta}$ are:

$$\left.\frac{\partial \tilde{\eta}}{\partial x}\right|_{y = 0} = \left.\frac{\partial \tilde{\eta}}{\partial x}\right|_{y = 1} = \left.\frac{\partial \tilde{\eta}}{\partial y}\right|_{y = 1} = 0$$

and

$$\left.\frac{\partial \tilde{\eta}}{\partial y}\right|_{y = 0} = - \cos x$$

The solution for $\tilde{\eta}$ is

$$\tilde{\eta} = \text{const} - \cos x \; y(1 - y)^2$$

Substituting $\tilde{\eta}$ into the equations for the approximate velocity profiles we get

$$\bar{u} \approx 2\omega r_1 \Delta\delta \frac{\partial \tilde{\eta}}{\partial x} = 2\omega r_1 \Delta\delta [y(1 - y)^2] \sin x$$

and

$$\bar{v} \approx \omega r_1 (1 - y) - 2\omega r_1 \delta \left(\frac{\partial \tilde{\eta}}{\partial y} + (1 - y) \cos x\right) =$$

$$\omega r_1 (1 - y) - 2\omega r_1 \delta [3y(1 - y)] \cos x$$

as the classical limit.

The velocity profiles in the classical limit predict a radial perturbation proportional to $\sin x$ and a circumferential perturbation proportional to $\cos x$. Since $x = 0$ and $x = \pi$ coincide respectively with points of minimum and maximum film thickness and $x = \pi/2$ and $x = 3\pi/2$ coincide with points at which the film thickness is changing most rapidly, the perturbation profiles in the classical limit can be viewed as being "in-phase" with the film thickness. Another way of stating this is that the classical perturbation profiles are "in-phase" with the distortion of the boundaries from axial symmetry. For all practical purposes the classical perturbation profiles coincide with the curves for $Re^* = 10^{-3}$ in figures 2 and 4. For that matter, they also coincide with the $Re^* = 10^{-3}$ curves in figures 3 and 5, because the radial component proportional to $\cos x$ and the circumferential component proportional to $\sin x$ vanish as the modified Reynolds number takes on classically small values. If the curves in figures 2 and 4 are to be interpreted as components "in-phase" with the boundary distortion, then the curves in figures 3 and 5 can be interpreted as components with a 90° phase shift or "out-of-phase" components.

Now let us consider the parametric changes in the profiles. First, we note that the "in-phase" components in figures 2 and 4 retain their general shape and exhibit at most a 20% deviation in magnitude from $Re^* = 10^{-3}$ to $Re^* = 10^2$. In short, the "in-phase" components are rather insensitive to the level of inertia. However, this is not so for the "out-of-phase" components. Not only is the growth of the "out-of-phase" components rapid after Re^* exceeds unity, but the magnitude of the "out-of-phase" components becomes a significant fraction of the "in-phase" magnitudes for $Re^* > 10$. Therefore, the most significant phenomena accompanying an increase in Re^* past unity are the appearance and rapid growth of velocity components which are 90° out-of-phase with the geometry of the physical flow region.

Admitting the existence and the rapid growth of the "out-of-phase" components, what are the consequences? First, the circumferential pressure distribution in the lubricant film is primarily dependent on the eccentricity perturbation of the circumferential velocity component. Therefore, the circumferential pressure distribution will exhibit a rapid change in shape as Re^* exceeds unity. If the circumferential pressure distribution changes shape, the load vector of the bearing will change direction and the bearing attitude angle will change. Secondly, the existence of the "out-of-phase" components influences the stability of the lubricant flow. However, the importance of the phase shift in stability considerations must be left for the future because the entire problem of flow stability for Reynolds numbers near the Taylor boundary has only been approached from a rather qualitative standpoint[5,6]. Therefore, since attitude angle strongly influences rotor stability and phase relations between velocity components has a strong influence on hydrodynamic stability, the "out-of-phase" components are primarily of importance in stability considerations. How much importance has yet to be determined.

CONCLUSIONS

Within the limits of validity of the assumptions that the eccentricity is small and the unperturbed flow is simply rotating Couette flow, the following conclusions may be drawn.

(1) Shaft orbiting does not have a profound effect on the fluid mechanic behavior of the lubricant film.

(2) The most significant changes in the velocity profiles associated with increasing values of the modified Reynolds number (but still less than the critical value for Taylor vortices) is the existence of components which are 90° out-of-phase with the film thickness.

(3) The major consequences of the "out-of-phase" components are yet to be determined. However, it is anticipated that they will be important in considerations of shaft and rotor dynamics and the fluid-dynamic stability of the lubricant film.

The conclusions of the present analysis indicate the need for future work in two directions. First, rotor dynamic calculations should be carried out to determine the effect of the "out-of-phase" velocity components on rotor stability. Secondly, the hydrodynamic stability of the lubricant film near the Taylor boundary should be investigated using at least the first order perturbation profiles. The second task will be much more difficult than the first.

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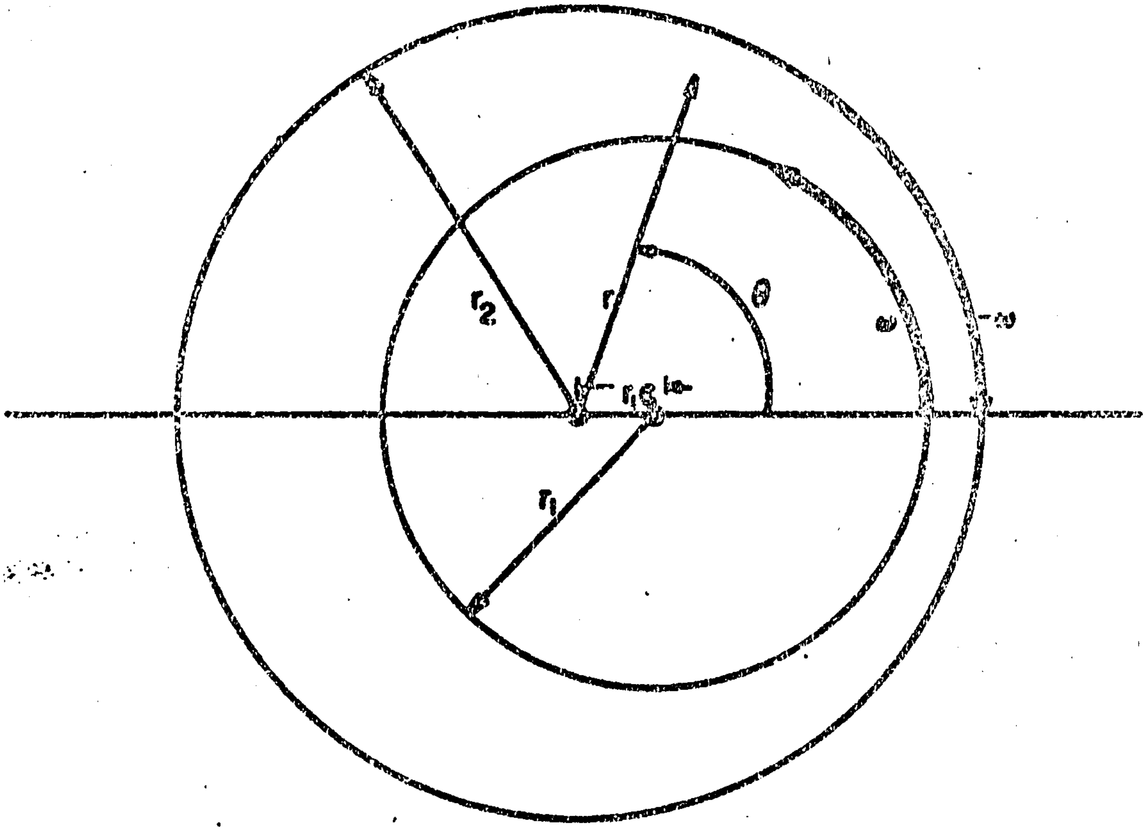
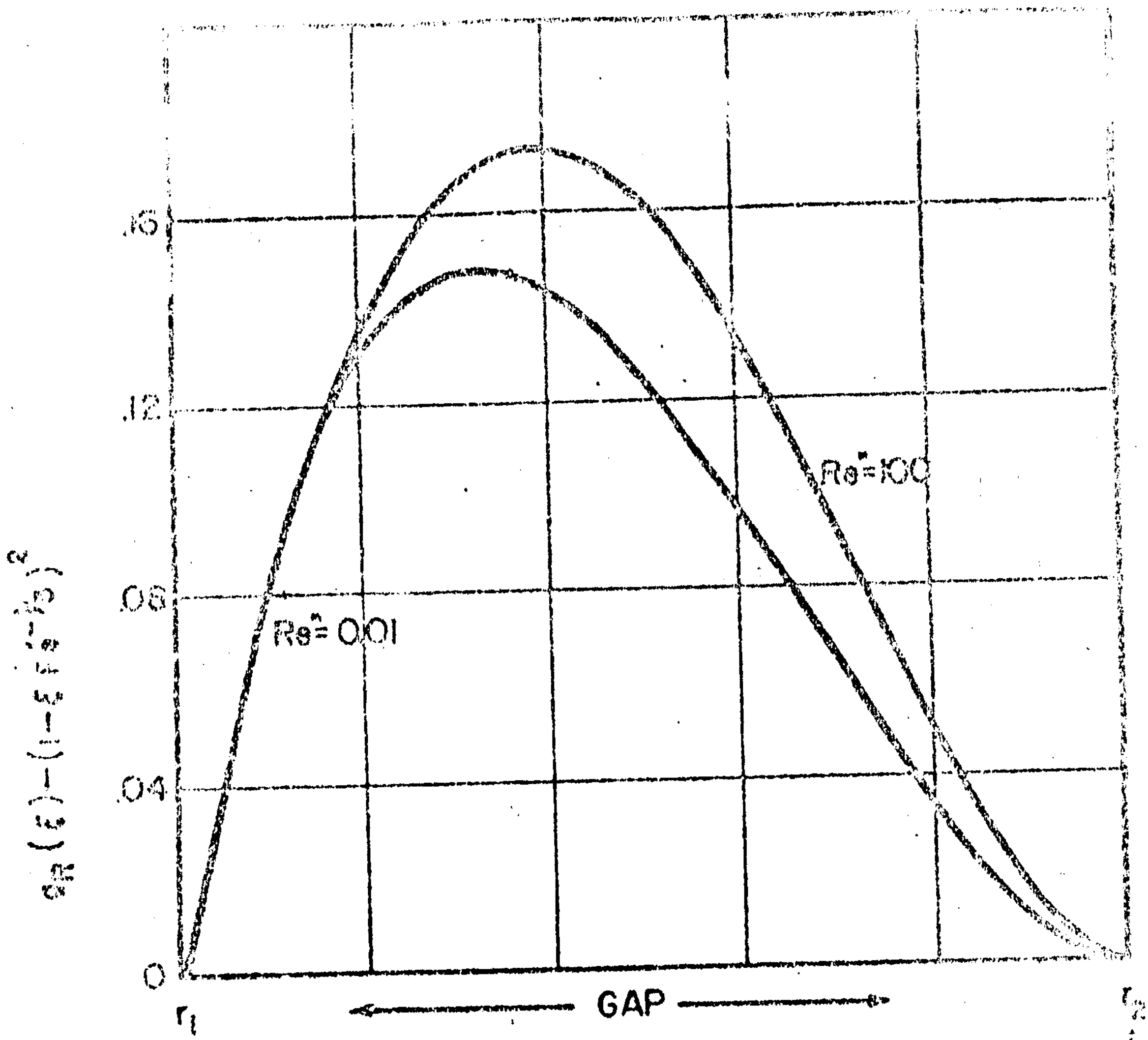
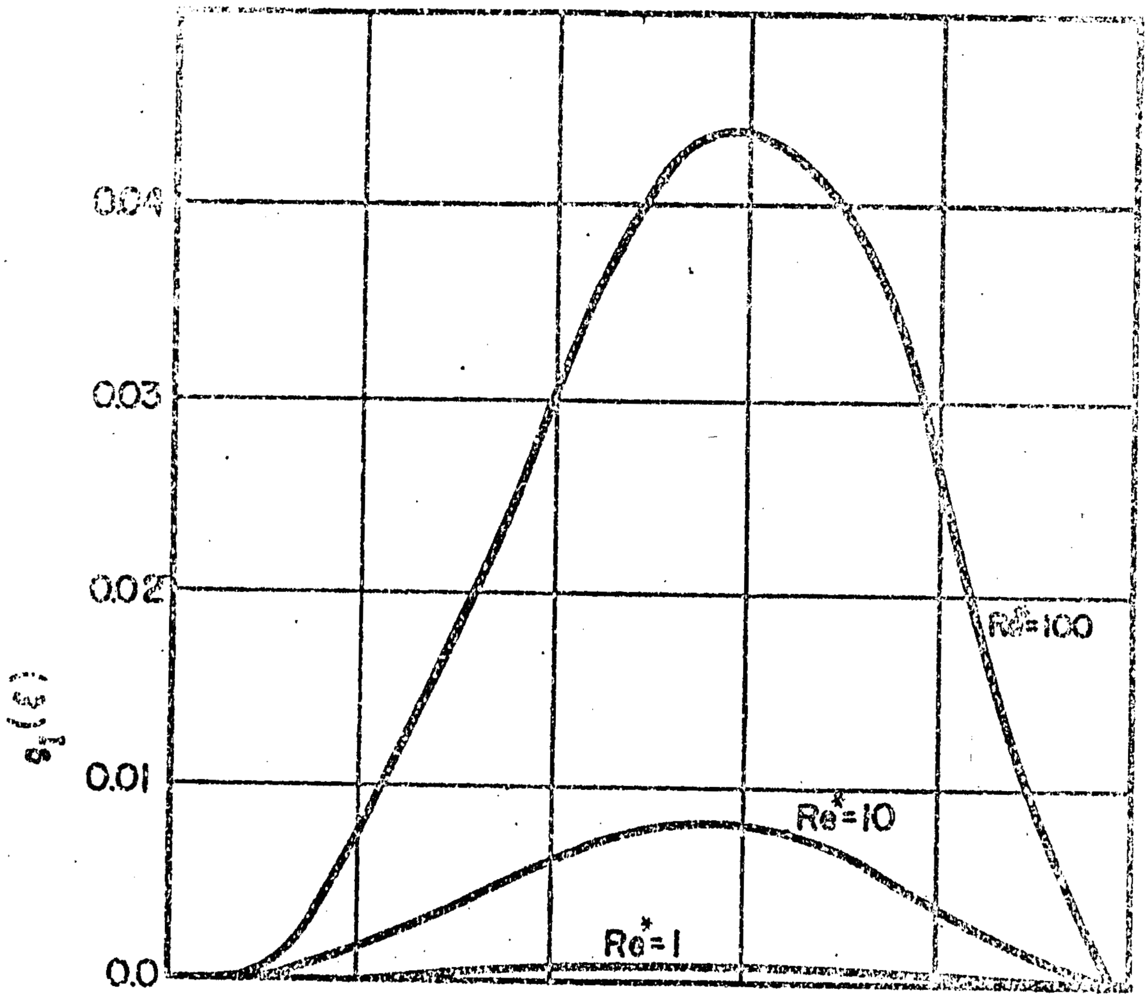


FIGURE 1 — BEARING REPRESENTATION



$Re^*=0.01$	0.12800	0.14400	0.09600	0.03200
$Re^*=0.1$	0.12800	0.14400	0.09600	0.03200
$Re^*=1.0$	0.12800	0.14401	0.09601	0.03200
$Re^*=10$	0.12817	0.14464	0.09676	0.03234
$Re^*=100$	0.13479	0.17273	0.13287	0.04912
$Re^*=10^{-3}$, $Re^*\rightarrow 0$	0.12000	0.14400	0.09600	0.03200

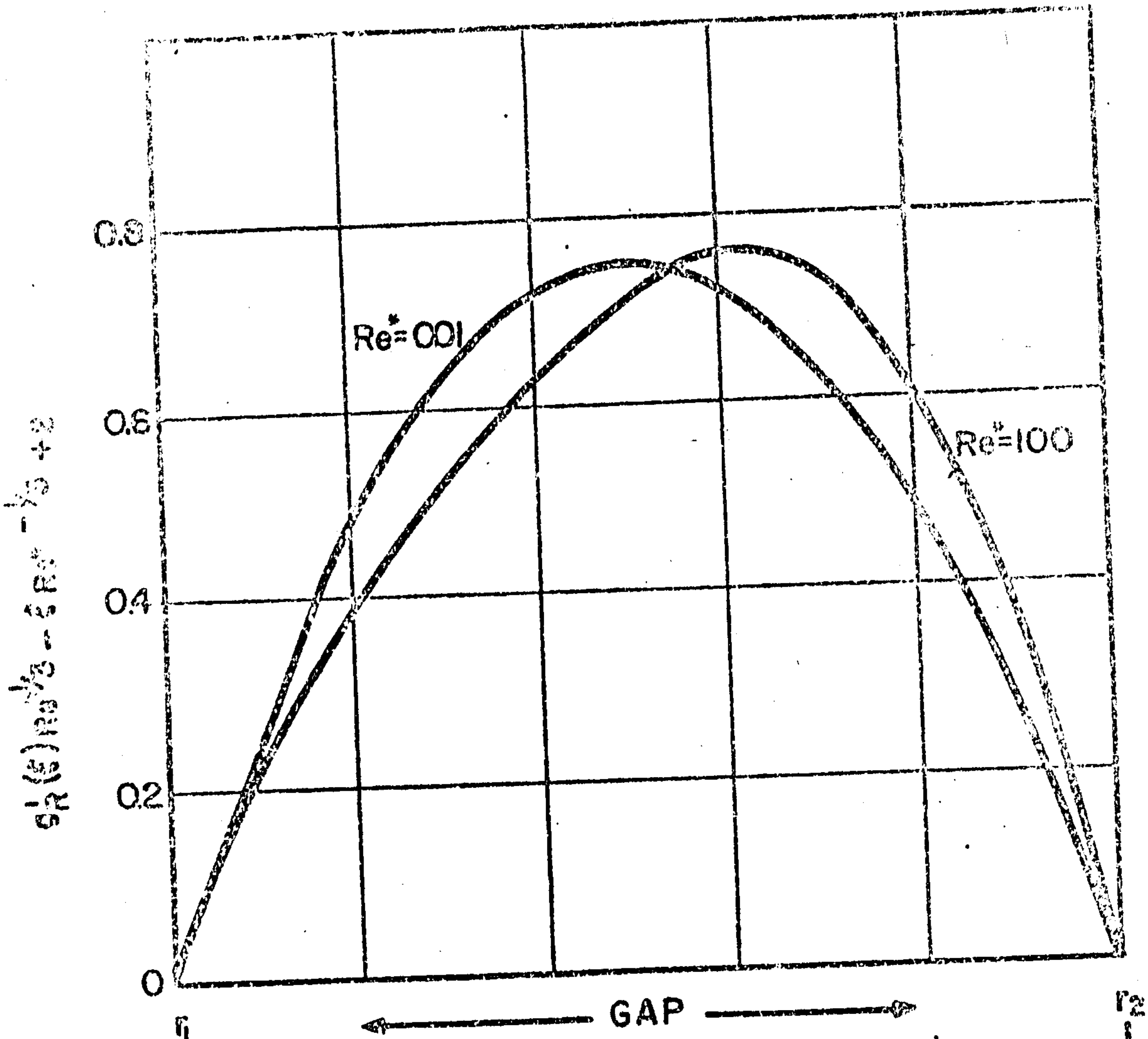
FIGURE 2 – FIRST ORDER RADIAL "IN PHASE" COMPONENT WITH TABULATED VALUES FOR FIXED ECCENTRICITY



r_1 ← GAP → r_2

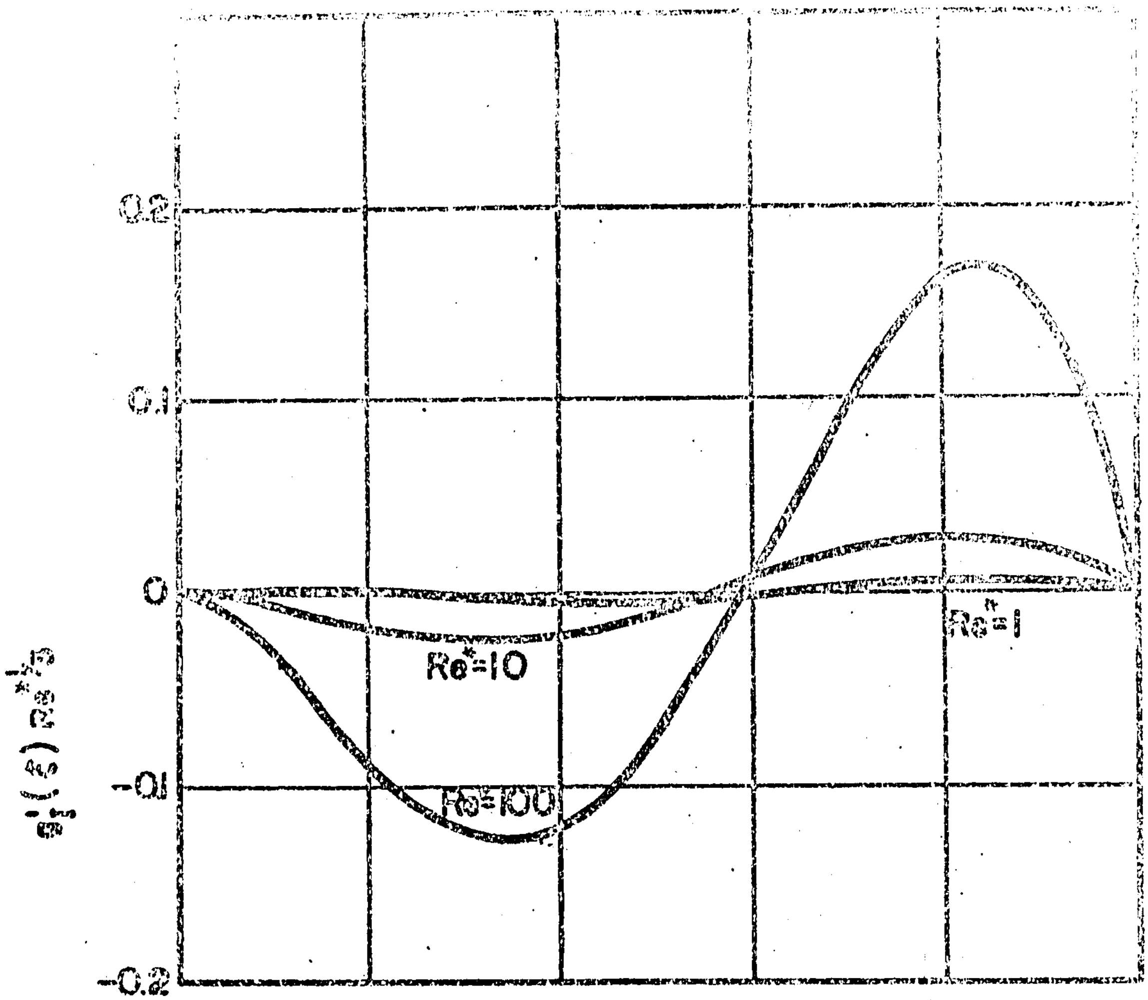
$Re^* = 0.01$	0.000000	0.00001	0.00001	0.00000
$Re^* = 0.1$	0.000002	0.00006	0.00008	0.00004
$Re^* = 1.0$	0.00015	0.00061	0.00081	0.00041
$Re^* = 10$	0.00152	0.00608	0.00799	0.00407
$Re^* = 100$	0.00687	0.03022	0.00390	0.00506
$Re^* > 10^{-3}$, $Re^* \to 0$	0.00000	0.00000	0.00000	0.00000

FIGURE 3—FIRST ORDER RADIAL "OUT OF PHASE" COMPONENT WITH TABULATED VALUES FOR FIXED ECCENTRICITY



Re* = 0.01	0.48000	0.72000	0.72000	0.48000
Re* = 0.1	0.48000	0.72000	0.72000	0.48000
Re* = 1	0.74988	0.71998	0.72001	0.48003
Re* = 10	0.47805	0.71798	0.72101	0.47263
Re* = 100	0.39565	0.61610	0.75712	0.61113
Re* = 10^{-3}, Re* → 0	0.48000	0.72000	0.72000	0.48000

FIGURE 4 — FIRST ORDER CIRCUMFERENTIAL "IN PHASE" COMPONENT WITH TABULATED VALUES FOR FIXED ECCENTRICITY



r_1 ← GAP → r_2

$Re^* = 0.01$	-0.00002	-0.00002	-0.00001	-0.00003
$Re^* = 0.1$	-0.00018	-0.00022	-0.00006	-0.00029
$Re^* = 1$	-0.00184	-0.00218	-0.00058	-0.00290
$Re^* = 10$	-0.00814	-0.02150	-0.00563	-0.02877
$Re^* = 100$	-0.08858	-0.11938	-0.00566	-0.16387
$Re^* = 10^{-3}$, $Re^* = 0$	0.00000	0.00000	0.00000	0.00000

FIGURE 5—FIRST ORDER CIRCUMFERENTIAL "OUT OF PHASE" COMPONENT WITH PARTIAL TABULATION FOR FIXED ECCENTRICITY